BMF and Parallelism

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OUTLINE

- BMF
- PARALLELISM
- PROOF OF CORRECTNESS

The Bird-Meertens Formalism

Notations

- Lists ($[\alpha]$: list of elements with the type α)
  - Definition
    - **[]** : Empty list
    - **[x]** : list with one element
    - **a ++ b** : concatenation of **a** and **b**
  - Concatenation
    - **[x] ++ xs = x :: xs**
    - **[a] ++ [b] ++ [c] = [a; b; c]**

Definition

- Binary trees :

  type BTree α β := ***Leaf*** $(n : \alpha)$ | ***Node*** $(n : \beta)$ *(*l*: BTree α β) (*r*: BTree α β)*

- Rose Tree :

  type RTree α := ***RNode*** $(n : \alpha)$ *[RTree α β]*

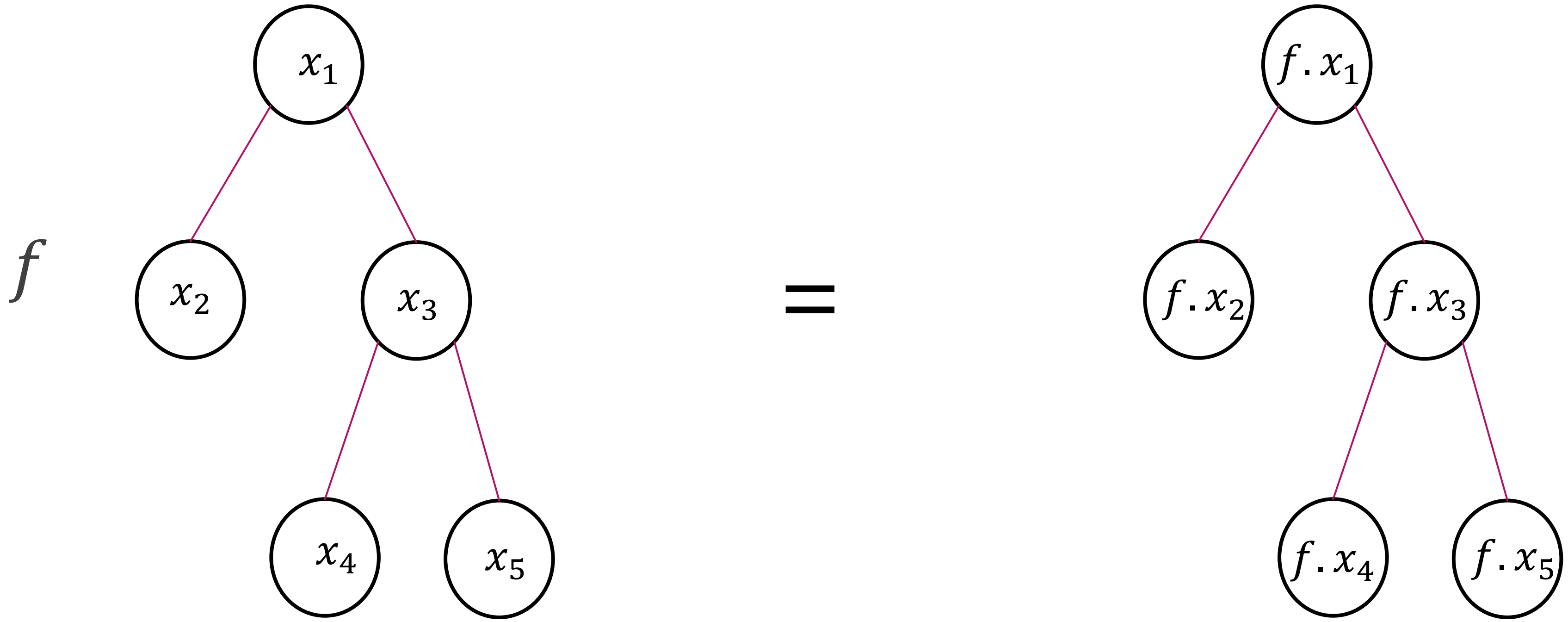
- *r2b : RTree α -> BTree α α*
- *b2r : BTree α α -> RTree α*

Primitives (Lists)

- Map : $f * [a_1, \ldots, a_n] \quad = \quad [f\ a_1, \ldots, f\ a_n]$
- Reduce : $\oplus/[a_1, \ldots, a_n] \quad = \quad a_1 \oplus \ldots \oplus a_n$
- Scan : $\oplus \not\!/\!\!/_e [a_1, \ldots, a_n] \quad = \quad [e, e \oplus a_1, e \oplus a_1 \oplus \ldots \oplus a_n]$

Primitives (Trees)

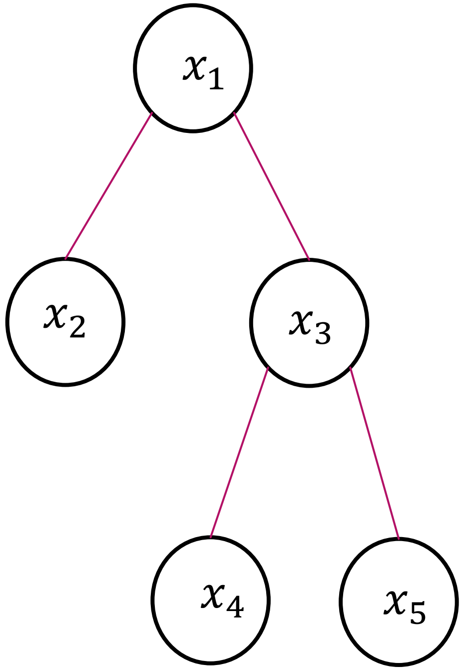
- Map :



Primitives (Trees)

- Reduce :

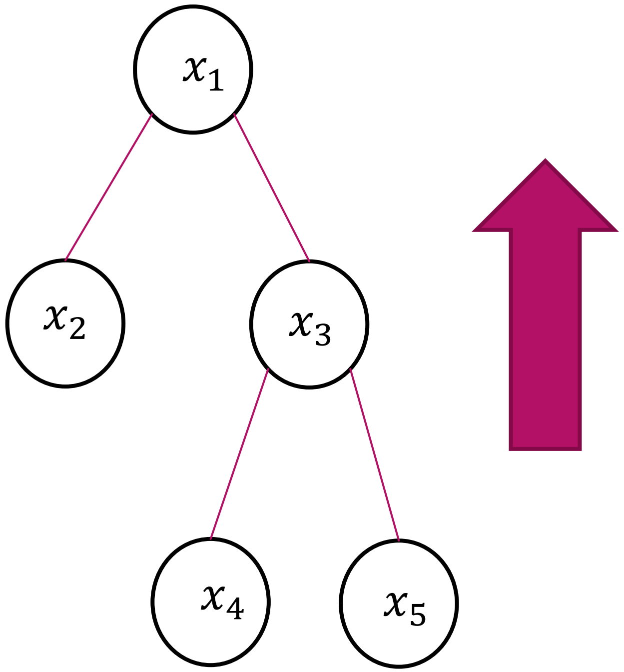
$\oplus /$



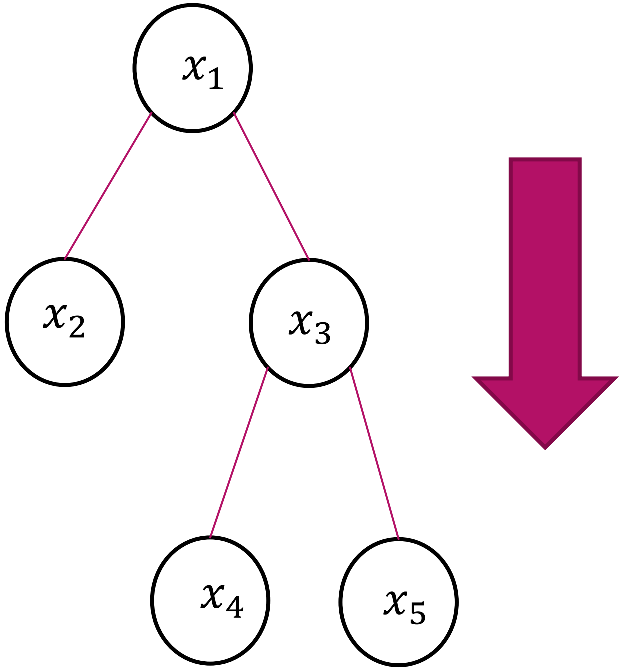
$$= \quad x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$$

Primitives (Trees)

- Upward accumulation :



- Downard accumulation :



Parallelization

Distribution List

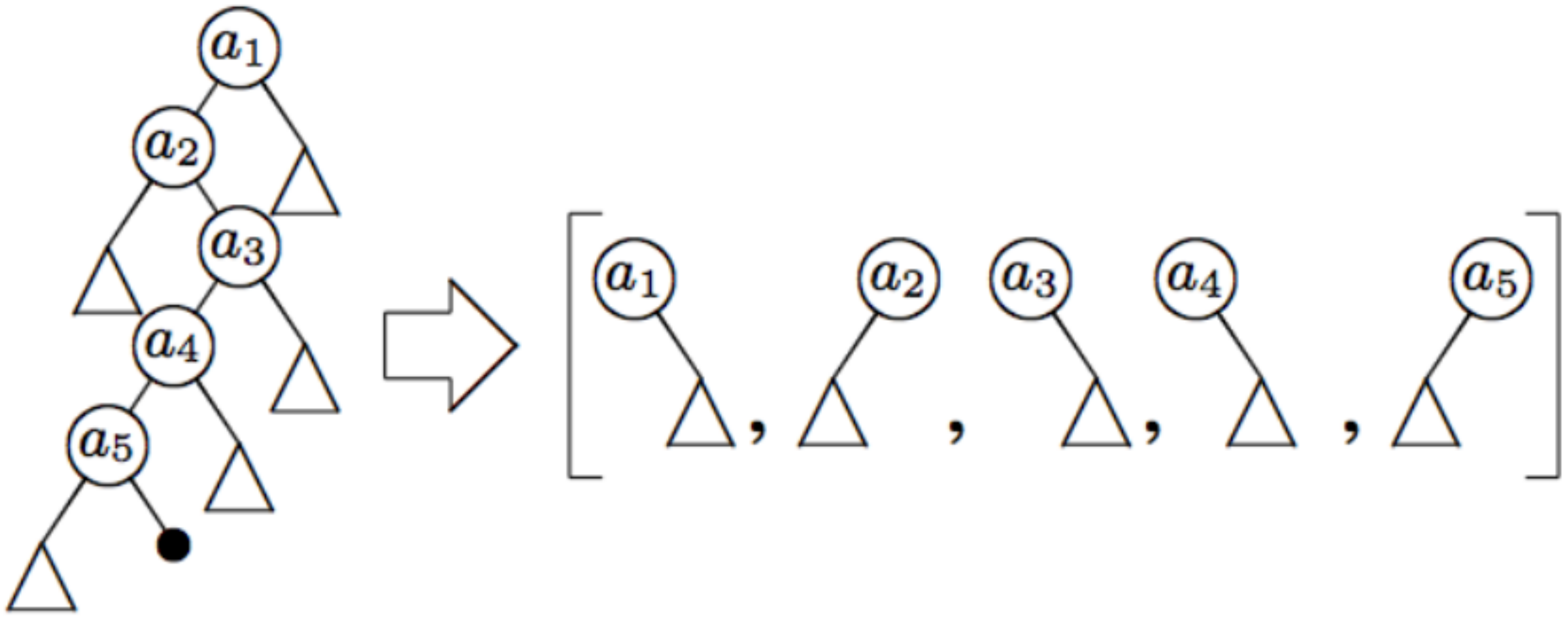
INPUT $[x_1, \dots, x_k,\ x_{k+1}, \dots, x_l,\ x_{l+1}\ \dots, x_n]$

DISTRIBUTION $[x_1, \dots, x_k]$ $[x_{k+1}, \dots, x_l]$ $[x_{l+1}\ \dots, x_n]$

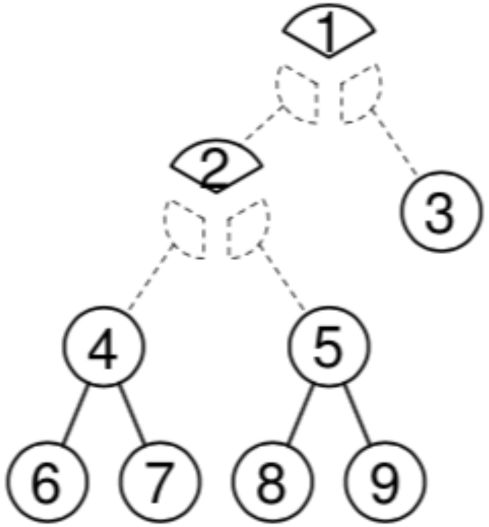
COMPUTATION $f\,[x_1, \dots, x_k]$ $f\,[x_{k+1}, \dots, x_l]$ $f\,[x_{l+1}\ \dots, x_n]$

OUTPUT $y_1 \oplus y_2 \oplus y_3$

Serialization Tree



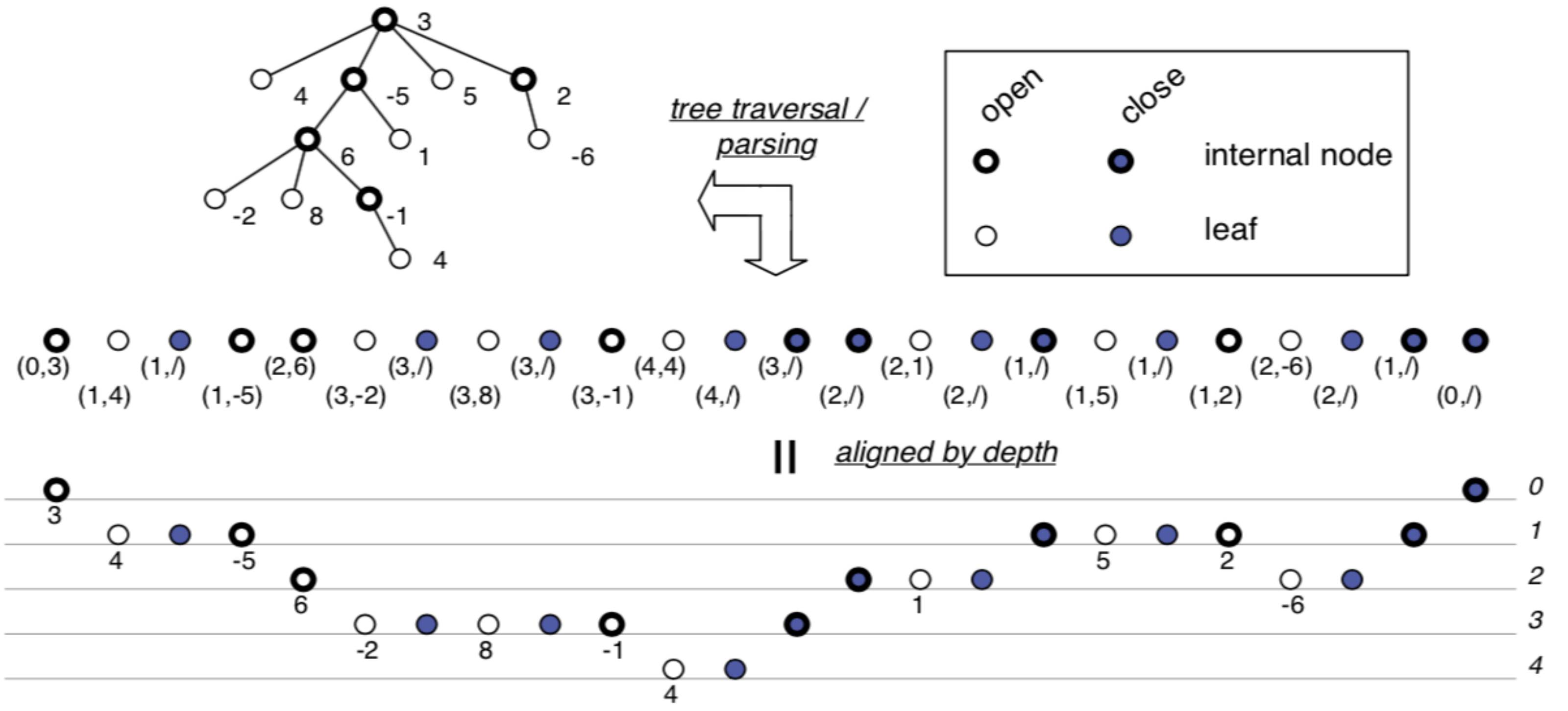
Serialization Tree



$$gt = [\square^{\mathsf{N}}, \square^{\mathsf{N}}, \square^{\mathsf{L}}, \square^{\mathsf{L}}, \square^{\mathsf{L}}]$$

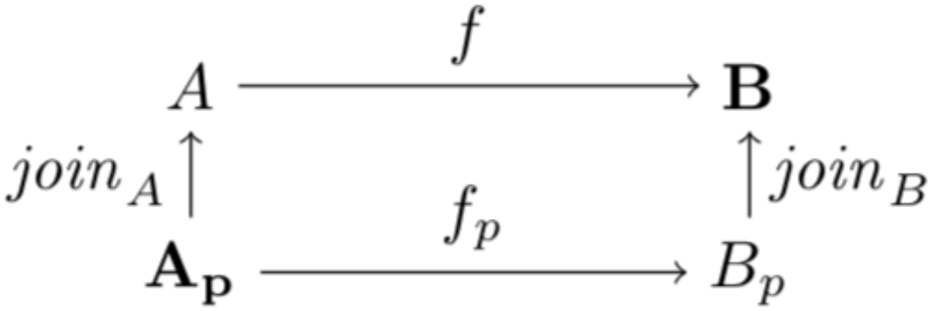
$$segs = [\,[\,1^{\mathsf{C}}\,],\; [\,2^{\mathsf{C}}\,],\; [\,4^{\mathsf{N}}, 6^{\mathsf{L}}, 7^{\mathsf{L}}\,],\; [\,5^{\mathsf{N}}, 8^{\mathsf{L}}, 9^{\mathsf{L}}\,],\; [\,3^{\mathsf{L}}\,]\,]$$

Distribution



Correctness

PROOF ?



To prove: $f_p \circ join_B = join_A \circ f$

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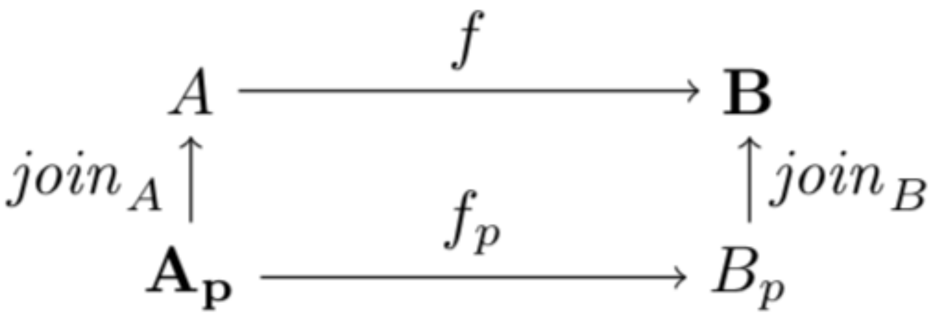

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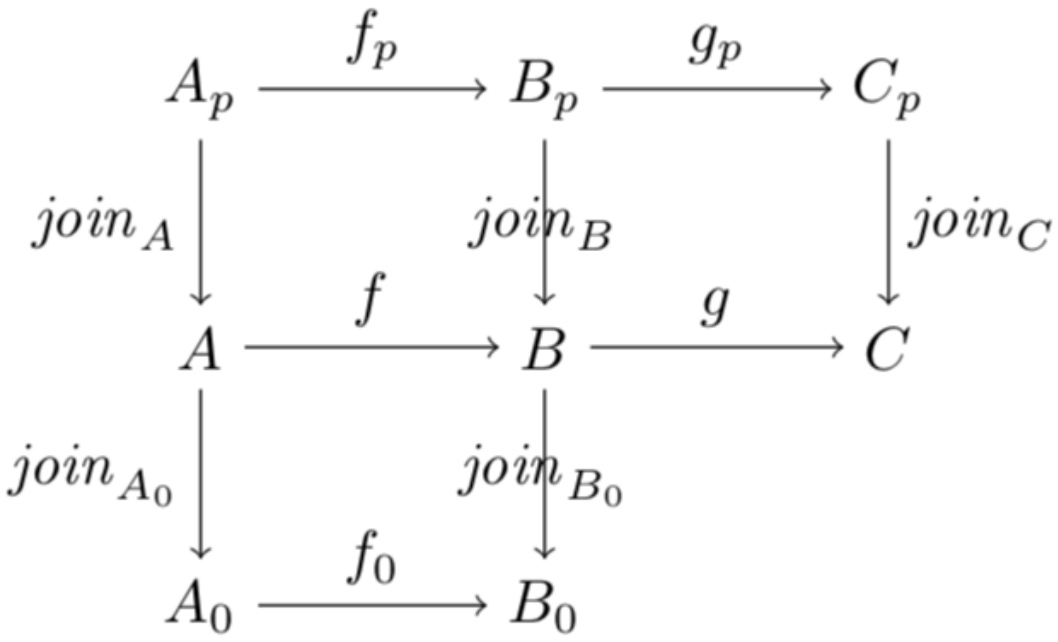

To prove: $f_p \circ join_B \circ join_{B_0} = join_A \circ join_{A_0} \circ f_0$

$f_p \circ join_B = join_A \circ f$

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